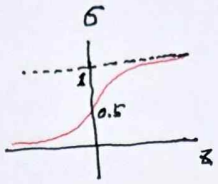
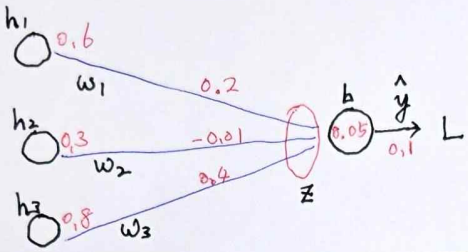
[Error Back Propagation]

- σ (sigmoid 함수)

$$\frac{1}{1+e^{-z}}$$





정답(y)은 1이라고 가정,

$z = w_1h_1 + w_2h_2 + w_3h_3 + b$

$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}} = 0.1$

학습률 $(\eta) = 0.1$로 가정

1) $Loss = \frac{(정답 - 출력값)^2}{2} = \frac{(1-0.1)^2}{2} = 0.405$

2) $\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z} \times \frac{\partial z}{\partial w_i}$

$\rightarrow \left[\frac{(y-\hat{y})^2}{2}\right]' = \hat{y} - y = 0.1 - 1 = \boxed{-0.9}$

$\frac{\partial \sigma(z)}{\partial z} = \underbrace{\hat{y}\cdot(1-\hat{y})}_{\text{미분공식}} = 0.1(1-0.1) = \boxed{0.09}$

$\frac{\partial (z)}{\partial w_i} \Rightarrow \frac{w_1h_1 + w_2h_2 + w_3h_3 + b}{\partial w_i}$

$\Rightarrow \frac{\partial z}{\partial w_1} = h_1 \underset{0.6}{}, \quad \frac{\partial z}{\partial w_2} = h_2 \underset{0.3}{}, \quad \frac{\partial z}{\partial w_3} = h_3 \underset{0.8}{}, \quad \frac{\partial z}{\partial b} = 1$

$\frac{\partial L}{\partial w_i} = \overbrace{(-0.9) \times (0.09)}^{-0.081} \cdot \frac{\partial z}{\partial w_i}$

$\frac{\partial L}{\partial w_1} = -0.081 \times 0.6 = -0.0486$

$\frac{\partial L}{\partial w_2} = -0.081 \times 0.3 = -0.0243$

$\frac{\partial L}{\partial w_3} = -0.081 \times 0.8 = -0.0648$

$\frac{\partial L}{\partial b} = -0.081 \times 1 = -0.081$

$$w_i^{new} = w_i^{old} - \eta \cdot \frac{\partial L}{\partial w_i}$$

$$\begin{cases} w_1^{new} = 0.2 - 0.1 \times (-0.0486) = 0.20486 \\ w_2^{new} = -0.1 - 0.1 \times (-0.0243) = -0.09757 \\ w_3^{new} = 0.4 - 0.1 \times (-0.0648) = 0.40648 \end{cases}$$

$$[\, b^{new} = 0.65 - 0.1 \times (-0.081) = 0.0581$$

$$\Rightarrow \underline{z^{new}} = w_1^{new} \cdot h_1 + w_2^{new} \cdot h_2 + w_3^{new} \cdot h_3 + b$$

$$= 0.20486 \cdot 0.6 + (-0.09757) \cdot 0.3 + 0.40648 \cdot 0.8 + 0.0581$$

$$= 0.476929$$

$$\boxed{\hat{y}^{new}} = \sigma(\underline{z^{new}}) = \sigma(0.476929) \approx 0.617$$

한번의 (가중치 + bias) 업데이트로,

출력값이 0.1 → 0.617로 조정됨.

[문제] 히든노드가 4개이고, 정답이 0이고 현재 가중치 등의 갑이 다음과 같은경우 백프로퍼게이션으로 각 업데이트값과 출력값을 구하시오

h1=0.2, h2=0.5, h3=0.7, h4=0.4
w1=0.3, w2=-0.2, w3=0.4, w4=0.1, b=0.05
현재 출력값 = 0.8
학습율=0.1